

## Exercises in 3D Computer Vision

### Exercise 5      **Transformations in Projective Space**

In the lecture you have been introduced to transformations of points and lines as well as to the hierarchy of transformations.

- Explain algebraically why lines are transformed as  $l' = H^{-T}l$  if points are transformed as  $x' = Hx$ .
- Given are two 2D points  $x = (1, 2, 3)^T$  and  $y = (1, 2, 1)^T$ . Join the two points to a line  $l$ . Make a transformation that translates a point 1 unit in x-direction and rotates it about  $90^\circ$ . Apply the transformation to  $x$  and  $y$  and its inverse transposed to  $l$ . Now, join the transformed points  $x'$  and  $y'$  to  $l'$ .

### Exercise 6      **2D Image Transformations in MatLab**

In the lecture you have learned how to transform single points in 2D using similarities, affinities, or projectivities. Now, we want to apply these transformations not only to one point but to an entire image.

- What problem occurs when dealing with rastered image coordinates? How can it be dealt with?
- An image in MatLab is an  $n \times m$  matrix with color or intensity values as its entries. The height is the number of rows ( $n$ ) and the width is the number of columns ( $m$ ). For image transformations we are at first not interested in the color values but in the coordinates. For applying transformations to the coordinates of an image in MatLab, one has to create a matrix with all coordinates as its entries. Form a matrix in MatLab where each coordinate of an image with size  $7 \times 7$  pixels is a column with three entries (homogeneous coordinates). Apply your method to an image with  $256 \times 256$  pixels.
- Once you have created the coordinate-matrix you can apply the transformation. How do you get an image from the transformed coordinates?
- (H) Write a method `imTrans` in MatLab that takes two arguments, an image and a transformation, and returns the transformed image.

### Exercise 7      **Conics and Quadrics**

Conics as well as quadrics are quadratic forms and can be expressed as  $x^T A x = 0$ , where  $A$  is the matrix of the quadratic form, which is symmetric. Under the projective transformation  $H$  taking a point  $x$  to  $x' = Hx$  a conic  $C$  transforms to  $C' = H^{-T} C H^{-1}$ .

- Show that a conic  $C$  and two points,  $x_1$ , and  $x_2$ , in general position have the invariant

$$Inv = \frac{(x_1^T C x_2)^2}{(x_1^T C x_1)(x_2^T C x_2)}.$$

- b) A similarity transformation leaves two points invariant - the circular points  $I = (1, i, 0)^T$  and  $J = (1, -i, 0)^T$ . Show it.
- c) Proof that the conic dual to the circular points,  $C_\infty^* = IJ^T + JI^T$  is also invariant under a similarity transformation.
- d) If two lines  $l$  and  $m$  are given in  $\mathbb{R}^2$ , how can the angle between the two be determined? Develop a formula that determines the angle between the two lines in  $\mathbb{P}^2$  and is invariant to projective transformations.

**Exercise 8 (H) Euclidian Transformations**

For the following excercises we take the point set  $p = [0 \ 0; 1 \ 0; 1 \ 1; 0 \ 1; .5 \ 1.6; 1 \ 1; 0 \ 0; 0 \ 1; 1 \ 0]'$   
 These can be changed to homogeneous coordinates by  $p\_h = [p; \text{ones}(1, 9)]$

Both can be plotted by `plot(p(1, :), p(2, :)); axis equal`

Note that this is just possible because the  $w$ -value of a point  $[xyw]^T$  is normalized to 1.

Just to remind you, if  $\mathbf{R}$  is transformation matrix and  $\mathbf{P}$  is a matrix that was generated by concatenated vectors  $\mathbf{R} \cdot \mathbf{P}$  does the same as transforming the points first and concatenating them after.

All of the exercises must be calculated in homogeneous coordinates.

- a) Create a transformation matrix  $\mathbf{M}_1$  that translates the points by  $\mathbf{t} = (3, -4)^T$
- b) Create a transformation matrix  $\mathbf{M}_2$  that rotates the points by 20 degrees. Mind that the function `sin()` just accepts rad values.
- c) Create a transformation matrix  $\mathbf{M}_3$  by multiplication that applies the rotation first and the translation after.
- d) Create a transformation matrix  $\mathbf{M}_4$  by multiplication that applies the translation first and the rotation after.
- e) Express  $\mathbf{M}_3$  and  $\mathbf{M}_4$  by the rotation matrix  $\mathbf{R}$ , translation vector  $\mathbf{t}$ . Do this calculation manually.

**Exercise 9 (H) Affine Transformations**

- a) Apply the matrix  $\begin{pmatrix} 1 & 2 & 0 \\ 0.5 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$  to the set of points.
- b) How would the matrix look like if the bottom right point of the house would be projected to  $(2, -.5)^T$  and the left top point to  $(-1, 1)^T$ ?

**Exercise 10 (H) Projective Transformations**

- a) Apply the matrix  $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0.3 & 0.5 & 1 \end{pmatrix}$  and plot it.
- b) Write the function `plotHomo(p)` that normalizes each point before showing and plots the points after.
- c) Apply the matrix  $\begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$  and plot it. Can you explain what you see?